UNIVERSITÄT DES SAARLANDES
FACHRICHTUNG MATHEMATIK
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Assignments for the lecture
Potential Theory in the Complex Plane
Summer term 2020

Assignment 3 B
for the tutorial on *Monday, June 22, 1:00 pm*

Exercise 1.

(i) Let X be a topological space and let $f_1, f_2 : X \to [-\infty, +\infty)$ be upper semicontinuous functions. Show that $\max\{f_1, f_2\}$ and $\alpha_1 f_1 + \alpha_2 f_2$ for all $\alpha_1, \alpha_2 \geq 0$ are upper semicontinuous.

(ii) Let $\emptyset \neq \Omega \subseteq \mathbb{R}^N$ be open and consider $s_1, s_2 \in S(\Omega)$. Show that $\max\{s_1, s_2\} \in S(\Omega)$ and $\alpha_1 s_1 + \alpha_2 s_2 \in S(\Omega)$ for all $\alpha_1, \alpha_2 \geq 0$.

(The former verifies in particular that $s^+ := \max\{s, 0\}$ is subharmonic on Ω whenever s is subharmonic; this was noticed in Remark 5.3 (ii) of the lecture. The latter property means that $S(\Omega)$ is a *cone*.)

Exercise 2. Let $\emptyset \neq \Omega \subseteq \mathbb{R}^N$ be open and connected. Consider a sequence $(s_n)_{n=1}^\infty$ in $S(\Omega)$ which is pointwise decreasing (i.e., $s_n(x) \geq s_{n+1}(x)$ for every $x \in \Omega$ and all $n \in \mathbb{N}$). Show that the function $s : \Omega \to [-\infty, +\infty)$ defined by

$$s(x) := \inf_{n \in \mathbb{N}} s_n(x) \qquad \text{for all } x \in \Omega$$

either satisfies $s \equiv -\infty$ on Ω or is subharmonic on Ω.

Hint: In order to verify the subharmonic mean value property, use the monotone convergence theorem.