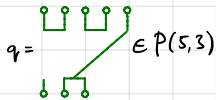
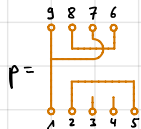
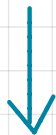
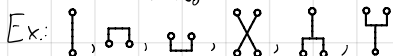


(set) partitions



$$P := (P(k, l))_{k, l \in \mathbb{N}_0}$$



C*-algebraic relations

$$p \in P(k, l) \quad \forall a_1, \dots, a_k, b_1, \dots, b_l \in \{1, \dots, n\}$$

$$R(p) := \sum_{i_1, \dots, i_k=1}^n \delta_{p(i_1, \dots, i_k)} u_{i_1, i_2} \dots u_{i_{k-1}, i_k} = \sum_{j_1, \dots, j_l=1}^n \delta_{p(j_1, \dots, j_l)} u_{j_1, j_2} \dots u_{j_{l-1}, j_l}$$

$$\& \quad u_{ij} = u_{ij}^* \quad i, j \in \{1, \dots, n\}$$

$$R(\text{block of size } 1): u_{ij} = u_{ij}$$

$$R(\text{block of size } 2): u_{ij}^* = u_{ji}$$

$$R(\text{block of size } 3): u_{ij}^* = u_{ji}$$

$$R(\text{block of size } 4): u_{ij}^* = u_{ji}$$

$$A_n(X) := C^*(1, u_{ij}, u_{ij}^* | u_{ij} = u_{ji}^* \& R(p) \forall p \in X)$$

non-orth. easy qu.groups?

$$P_0 := \{p \in P \mid p, p^* \in H_n(X) \Rightarrow \text{block of size } 1\}$$

$$\text{Prop: } p \in P_0, p, p^* \in H_n(X) \Rightarrow (A_n(X), u) \in Q_n^+ \text{ easy}$$

$$\text{Thm: } p \in P(0, l), p \neq \text{block of size } 1 \dots \text{block of size } l \text{ if } p \in P_0 \text{ if}$$

$$(a) \text{ } l \text{ odd, (b) } l \text{ even, } \nexists \text{ block of size } 1,$$

$$(c), (d), \dots \quad \text{3 block of size } \geq 3 \text{ in } p$$

$$Q: \text{block of size } 1 \in P_0? \text{block of size } 2 \in P_0?$$

"algebraic approach"

$$H_n(X) := \{p \in P \mid R(p) \text{ hold in } A_n(X)\} \cong X$$

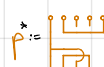
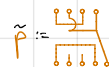
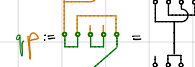
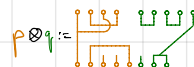
$$\text{Thm: } a) p, q \in H_n(X) \Rightarrow p \otimes q, qp, \tilde{p} \in H_n(X)$$

$$b) \text{block of size } 1, p \in H_n(X) \Rightarrow p^* \in H_n(X)$$

$$\text{Cor: } \mathcal{C} = \langle p_1, \dots, p_m \rangle \Rightarrow A_n(\mathcal{C}) = A_n(\{p_1, \dots, p_m, \text{block of size } 1\})$$

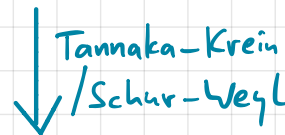
$$\text{Thm: } p \in P(0, l), p \neq \text{block of size } 1, p, p^* \in H_n(X) \Rightarrow (A_n(X), u) \text{ CMQG}$$

Category of partitions



$$\mathcal{C} = (\mathcal{C}(k, l))_{k, l \in \mathbb{N}_0} \text{ Krews category of partitions}$$

$$: \Leftarrow \mathcal{C} \text{ closed under above operations } \& \text{block of size } 1, \text{block of size } 2 \in \mathcal{C}$$



(easy) quantum group

$$G = (A, u) \text{ Compact matrix quantum group } [\text{Murphy 2003}]$$

$$\bullet A \text{ unital } C^* \text{-algebra generated by } u_{ij}, i, j = 1, \dots, n$$

$$\bullet u = (u_{ij}), \bar{u} = (u_{ij}^*) \text{ are invertible}$$

$$\bullet \Delta: A \rightarrow A \otimes A, u_{ij} \mapsto \sum_k u_{ik} \otimes u_{kj} \text{ *-hom.}$$

$$G \text{ "easy quantum group" } [\text{Banyaga, Speicher 2005}] : \Leftarrow A = A_n(\mathcal{C}), \mathcal{C} \text{ categ. of part}$$